Diversity and Delay-Limited Throughput Analysis for the Effective Cooperative ARQ Protocols with Opportunistic Distributed Space-Time Coding

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Abstract—The paper investigates effective protocols and the corresponding performance for cooperative automatic retransmission request (ARQ) that uses the decode-and-forward (DF) opportunistic distributed space-time coding (ODSTC). According to the analysis, the delay-limited throughput enjoys significant enhancement even using one or two active relays only for ARQs than using direct retransmissions. Besides, allowing the non-active relays to overhear the DSTC signal sent by active relays during ARQs also leads to a better throughput via ODSTC. Nevertheless, in the high or the low SNR regimes, simple schemes without overhearing may provide almost the same performance offered by a scheme with overhearing despite the much inferior diversities. These features may be further explored to enhance the overall system throughput for multiple access systems using only limited system resources.

Keywords—Cooperative ARQ, Opportunistic DSTC, Cooperative relaying.

I. INTRODUCTION

Cooperative communication has emerged as a new paradigm in wireless communications. Since the work of [1, 2], many cooperative ideas have been introduced to enhance the system capacity and/or transmission reliability, either through user cooperation or by signal relaying. Among them, a host of cooperative schemes have been proposed to exploit the spatial diversity via distributed space-time coding (DSTC), e.g. [2, 3]. The diversity order of the outage probability is shown to increase proportionally with the number of cooperative relays. Moreover, a full order of the cooperative diversity can be achieved even by using one out of a set of available relays opportunistically [4]. Motivated by the simplicity and effectiveness of opportunistic relaying, some more recent efforts have been made to investigate the opportunistic distributed beamforming [5] and DSTC [6].

In contrast to the rich results in the outage analysis for DSTC, the performance of automatic retransmission request (ARQ) is relatively less investigated for opportunistic DSTC (ODSTC). The average throughput of a cooperative hybrid-ARQ scheme has been reported in [3] for DSTC only. To study the effectiveness of ODSTC and the extra degrees of freedom via ARQs, in the beginning, the spatial and temporal diversities of cooperative ARQ via decode-and-forward (DF) ODSTC are investigated. Then, we analyze the delay- and outage-limited throughput to examine the effectiveness of each scheme at different SNRs. According to the analysis, allowing the non-active relays to overhear the DSTC signal sent by one or two active relays yields significant advantage on the delay-limited throughput. Nevertheless, in the high or low SNR regimes, simple schemes without overhearing may provide almost the same performance offered by overhearing, despite its inferior diversities. Thus, the need for a complex protocol with overhearing can be greatly reduced. Besides, the results also show that the throughput is seriously dominated by the acquisition delay for relaying, and the performance saturates with a large number of relays. These properties can be very helpful for multiple access systems as only a limited number of relays with higher channel access priority is needed to assign to each user.

II. PRELIMINARY

We consider a relay network with $M$ relays to help retransmit signals with DF ODSTC as shown in Fig. 1. In the beginning of a transmission, the source node broadcasts (BC) signal to all relays and the destination. The signal sent from the source and received at the destination and a relay node $m$ are modeled, respectively, as

$$r_{s,d} = \sqrt{P_{sd}}h_{s,d}x + n_d$$

(1)

$$r_{s,r_m} = \sqrt{P_{sr_m}}h_{s,r_m}x + n_m, \ m = 1,2 \cdots M$$

(2)

where $\sqrt{P_{sd}}$ is the baseband received power at the destination, and $\sqrt{P_{sr_m}}$ is the received power at the relays. To highlight the effects of ODSTC and to alleviate the complexity of analysis, we assume that the relays receive the same signal power from the source, while experience independent channel fades as the relays are distributed at different locations. The channel coefficients $h_{s,r_m}$ and $h_{s,d}$ are modeled as complex Gaussian random variables (RVs) with zero mean and variance one, denoted by $\sim C\mathcal{N}(0,1)$, and the noise $n_m$ and $n_d$ are also modeled as $\sim C\mathcal{N}(0,N_0)$. 

![Cooperative ARQ via opportunistic DSTC. The green nodes stand for the relays that have decoded successfully.](image-url)
If the signal is not successfully decoded at the destination, the relays that have decoded successfully will retransmit the data using DSTC, otherwise the source will re-broadcast the signal until either the destination or at least one relay is able to decode the signal. Throughout the paper, the set of relays that decode successfully is referred to as the decoding set and denoted by $S_D$. Again to focus on effective ARQ protocols for ODSTC, a perfect synchronization is assumed achieved among all relays. The assumption is realistic and necessary for cellular networks like WiMAX or LTE-A. Under these assumptions, the channels between the transmitting relays and the destination can be viewed as a multiple-input single-output (MISO) channel, thus the corresponding mutual information follows [7]

$$I_{rd} = \log \left( 1 + \frac{P_d}{N_0} \sum_{m \in S_D} |h_{r_m,d}|^2 \right)$$  

(3)

where $P_d$ is the received power at the destination for signals transmitted by relays in $S_D$, and $h_{r_m,d}$ is the channel between the relay $m$ and the destination and is $\sim \mathcal{CN}(0,1)$. Similarly, for channels between the transmit relays and a receive relay $r_\ell$, the mutual information is given by

$$I_{r_\ell} = \log \left( 1 + \frac{P_t}{N_0} \sum_{m \in S_D} |h_{r_m,r_\ell}|^2 \right), \ell \notin S_D.$$  

(4)

For convenience of analysis, we define $\rho \triangleq P_{sd}/N_0$ and have $P_{sr}/N_0 \triangleq \alpha \rho$, $P_{rd}/N_0 \triangleq \beta \rho$ and $P_{sr}/N_0 \triangleq \eta \rho$. Consequently, we have $|h_{r_m,d}|^2 \sim \mathcal{Exp}(\lambda)$, $|h_{r_m,r_\ell}|^2 \sim \mathcal{Exp}(\lambda_1)$, $\beta\rho|h_{r_m,d}|^2 \sim \mathcal{Exp}(\lambda_2)$ and $\eta \rho |h_{r_m,r_\ell}|^2 \sim \mathcal{Exp}(\lambda_3)$, for all $\lambda = N_0/P_{rd}$, $\lambda_1 \triangleq 1/(\alpha \rho)$, $\lambda_2 \triangleq 1/(\beta \rho)$ and $\lambda_3 \triangleq 1/(\eta \rho)$, respectively.

A. The Outage Probabilities for ODSTC

To proceed the analysis for ARQ protocols with ODSTC, we briefly review the ODSTC scheme below and give an exact expression for its outage probability.

Define $W \triangleq \rho |h_{r_m,d}|^2 \sim \mathcal{Exp}(\lambda)$, the outage probability for the direct source to destination channel link is given by

$$P_W(\delta_s) \triangleq P\{W < \delta_s\} = 1 - \exp\{-\lambda \delta_s\}$$  

(5)

where $\delta_s \triangleq (2^R - 1)$ and $R$ is the source rate in bits/sec/Hz. In cases of outage events, ODSTC opportunistically chooses at most $i$ relays in $S_D$ to perform STC [6]. To distinguish from ordinary DSTC and to signify the use of $i$ relays at most, we denote the protocol by ODSTCi. Define $X_m \triangleq \beta\rho|h_{r_m,d}|^2 \sim \mathcal{Exp}(\lambda)$, $m \in S_D$. Let $X_i \triangleq \{X_m|m \in S_D\}$. As ODSTCi chooses at most $i$ elements of $X$ that yield the highest capacity in (3), clearly min$\{i,D\} \leq |S_D|$, of the largest elements in $X$ will be chosen for STC. Therefore, sorting the elements of $X$ in the ascending order into $X' \triangleq \{X'_1, \ldots, X'_d\}$ such that $X'_k \geq X'_j$ if $k > j$, the outage probability of ODSTCi conditioned on $D$ is then given by

$$P_{O,D}(\delta|d) \triangleq P\left\{\delta_s \leq \max_{1 \leq d \leq i} X'_d < \delta\right\}$$  

(6)

where $\delta = 2^R - 1$ and $R$ is the code rate for DSTC. The outage probability (6) can be evaluated with a theorem quoted below from [8].

Theorem 1: [8] Let $\{X'_1 < \cdots < X'_Q\}$ be the order statistics from $Q$ i.i.d. exponential RVs with parameter $\nu$. Define

$$Z_{Q,q} = \sum_{q=1}^{Q} |X'_q - q|^{1/\nu}, 1 \leq q \leq Q.$$  

The complementary cumulative distributed function (CCDF) of $Z_{Q,q}$ is given by:

$$P\{Z_{Q,q} > z\} = \sum_{j=1}^{Q-q} a_j e^{-\frac{z}{j-1}} \left(1 + \frac{z}{j-1}\right)^{-1}$$  

$$\int_0^z e^{(b_j y)\left(\frac{y}{q-1}\right)} dy + \sum_{k=0}^{q-1} e^{-\frac{z}{k}} \left(\frac{z}{k}\right)^k k! \quad (7)$$

with $a_j \triangleq \frac{1}{j-1} \frac{(Q-q)^2-q-j}{Q-q-j}$ and $b_j \triangleq \frac{Q-q-j+1}{j-1}$. On the other hand, as $\alpha \beta \rho |h_{r_m,d}|^2 \sim \mathcal{Exp}(\lambda_1)$, the probability mass function (PMF) of $D$ is given by [2]

$$P_D(d) = C_M^d \left(e^{-\delta,\lambda_1}\right)^d \left(1 - e^{-\delta,\lambda_1}\right)^{M-d}.$$  

(8)

Based on the above results, for transmission followed by an ARQ using ODSTCi, the outage probability follows

$$P_i = P_W(\delta_s)^2 P_D(0) + P_W(\delta_s) \sum_{d=1}^{M} P_{O,D}(\delta|d) P_D(d).$$  

(9)

We note that for ODSTC1, the outage probability degenerates to the case of opportunistic relaying (OR) in [4], while for ODSTCM, it is equal to DSTC in [2].

III. THE CODING GAIN OF ODSTC

In this section, we characterize the relative SNR advantage of ODSTCM against the ODSTCi schemes. To alleviate the complexity of analysis, we investigate this problem in the high SNR regime. Since the diversity gain is defined as

$$\xi \triangleq \lim_{\rho \to \infty} \frac{\log P_i}{\log \rho},$$  

(10)

Based on the fact that the diversity orders of the ODSTCi and ODSTCM schemes are both $M + 1$ [2, 4], the diversity order of any ODSTCi scheme is also $M + 1$. Thus, the outage probability in the high SNR regime can be expressed as

$$P_i \approx G(i) \cdot \rho^{-(M+1)}$$  

(11)

where $G(i) \approx \lim_{\rho \to \infty} P_i/\rho^{(M+1)}$ and its properties are characterized in the following theorem.

Theorem 2: For DF-ODSTCi scheme, $i \in [1,M]$, operating at a rate $R$, we have

$$G(i) = \sum_{D=1}^{M} C_M^D \frac{\delta^M - D^M + 1}{\beta^D} \frac{\alpha^M - D^M}{\alpha^M} + \Delta(i)$$  

(12)

where

$$\Delta(i) = \sum_{D=i}^{M} \delta_s^{D-M+1} C_M^D \frac{\beta^D \alpha^M - b^M}{\beta^D \alpha^M} - \frac{1}{\beta^D}.$$  

(13)

Proof: The proof is omitted for space.

To characterize the SNR loss of the ODSTCi scheme against the SNR for the ODSTCM to achieve the same level of outage probability, $P_i$, at high SNR, we have $P' = G(i)\rho^{-\xi} = G(M)\rho^{-\xi}$ and define the SNR loss of a ODSTCi scheme against the ODSTCM, $\forall i \in [1,M]$, as

$$L_i \triangleq \log \left\{ \frac{\rho_i}{M+1} \right\} \log \left\{ G(M) \right\} \left(1 + \frac{\Delta(i)}{G(M)} \right).$$  

(14)
With some mathematical manipulations, we have two limiting values of $L_i$, summarized in the following corollary.

**Corollary 1:** For all $i \in [1, M],
\begin{align*}
L_i^{\alpha \to \infty} &= \frac{1}{M + 1} \log \left( \frac{M!}{\alpha_{i,1}(M-\alpha)} \right) \quad \text{as} \quad \alpha \to \infty. \\
L_i^{\beta \to \infty} &= 0 \quad \text{as} \quad \beta \to \infty.
\end{align*}

On the other hand,

**Proof:** The proof is omitted for space.

This shows that when $\alpha$ is large, $L_i$ becomes irrelevant of $\beta$ and $\delta$ and is only a function of $i$ and $M$. While when $\beta$ becomes large ODSTCi sever as good as ODSTCM does.

Fig. 2 shows the outage probability for $i = 1, 2, 3$ and ODSTCM, which is denoted by the purple-star curve and always performs best among all the schemes. Nevertheless, in this example, the black-circle curve, which corresponds to the case of $i = 3$, is almost undistinguishable from the purple one, ODSTCM scheme. It shows that the ODSTCi scheme performs closer to ODSTCM scheme with larger $i$. However, the SNR advantage for every scheme is also correlated with the links between the source, relays and destination. In Fig. 3, the red curve which corresponds to the case of $\alpha = \infty$, $\beta = 1$ tells the fact that when the link quality between the source and relay is very good, using more relays is better. But to the blue one, $\alpha = 8$, $\beta = 2$, using 4 relays is good enough in contrast to ODSTCM. Furthermore, for the case of $\alpha = 2$, $\beta = 8$, the outage performance is limited to the case of $i = 2$. The reason that the outage probability is limited by the case of $D = 0$ or 1. The energy on the relays that fail to decode the signal are not able to be used to help enhance the performance, leaving the option of using more relays ineffective.

**IV. COOPERATIVE ARQ PROTOCOLS USING ODSTC**

Based on ODSTCi presented in Section II, we study in this section the outage probabilities for three types of cooperative ARQ protocols. Each of them requires for different level of coordination between the source, relays and the destination.

As introduced earlier, the destination will issue ARQs to the source whenever $S_D = \emptyset$. Once $S_D \neq \emptyset$, the relays in $S_D$ will take over the retransmissions. Based on the ODSTCi relaying, three types of protocols can be used for ARQ. They are referred to as the Type-A, B and C respectively with increasing complexities in coordinations:

- **Type-A:** The destination chooses the best $i$ relays for ODSTCi when $S_D$ first turns nonempty and then continues to use these relays in the subsequent ARQs.
- **Type-B:** According to the channel strength in each ARQ, the destination re-chooses the $i$ relays from the same $S_D$ for retransmission.
- **Type-C:** The destination re-chooses the relays from the $S_D$ that may grow with ARQs when allowing the relays not in $S_D$ to overhear the DSTC signal from the active relays.

We study in the sequel the outage probabilities for the three types of ARQ protocols. For simplicity of presentation, the $n$-th round of ARQs is denoted by ARQ$n$, $n \in \mathbb{N}$, with the initial transmission from the source denoted by ARQ0.

For the Type-A protocol, the ARQs in fact involve two types of retransmissions: the ODSTCi for ARQ1 and the ordinary DSTC for ARQ$i$, $i \geq 2$ since the channel change independently in each ARQ. The outage probability for the DSTC using at most $i$ relays in $S_D$ is given by

$$
P_{Z_D}(\delta|D) = 1 - \sum_{\rho=0}^{i-1} e^{-\delta\lambda_2} y^{\rho} (\delta\lambda_2)^{-\rho}
$$

where $\delta D = \min\{i, D\}$. Together with $P_{O,D}(\delta|d)$ in (6), the outage probability of the Type-A ARQ can be expressed in a closed form summarized in the following proposition.

**Proposition 1:** Given $R$, $\epsilon$ and $M$, the outage probability after $n$ times Type-A ARQs is given by

$$
P_{A,n}(\rho) = P^n_W(\delta_{\rho})P^n_D(0) + P_W(\delta_s) \times \sum_{\rho=0}^{n} [P_W(\delta_s)P_D(0)]^{n-k} \sum_{d=1}^{M} P_{O,D}(\delta|d)P_{D}(d)P_{Z_D}^{k-1}(\delta|D)
$$

\forall i \in [1, M] \text{ with } P_{A,n}(0) \triangleq P_W(\delta_s) \text{ in (5).

Proof: } The proof is omitted for space.

For the Type-B protocol, the outage probability can be obtained by replacing $P_{Z_D}(\delta|D)$ in (17) with $P_{O,D}(\delta|d)$ due to the reselection process in every ARQ.
Corollary 2: Given $R$, $\varepsilon$ and $M$, the outage probability after $n$ times Type-B ARQs is given by
\[
P_{\text{B},i}(n) = P_{W}^{n+1}(\delta_i) P_{D}^{n}(0) + P_{W}(\delta_i)
\]
\[
= \sum_{k=1}^{n} [P_{W}(\delta_i) P_{D}(0)]^{n-k} \sum_{d=1}^{M} P_{\text{B},i,\ell}(\delta|d) P_{D}(d)
\]
\[\forall i \in [1, M] \text{ with } P_{\text{B},i}(0) = P_{W}(\delta_i).\]

As for a short summary, comparing with Type-A ARQ, apparently, Type-B ARQ requires all relays in $S_D$ to keep the decoded data for retransmission before the end of ARQs. However, checking $P_{\text{D},i}(\delta|d)$ in (18), one may soon find that the diversity order may often be dominated by the term $P_{\text{D},i,\ell}(\delta|d)$, leaving the overall diversity order remaining to be $M + n$. This may cause the Type-B scheme rather ineffective, taking into account the extra efforts for the re-selection of relays in ARQs.

As for the Type-C ARQ, the probabilities of outage events are more involved since the cardinality of $S_D$ may increase now with ARQs through overhearing. To account for the growing cardinality of $S_D$, we define some parameters below.

Definition 1: Let $D_0$ be the number of relays that are able to decode the signal sent by the source.

Definition 2: Let $D_n$ be the number of increasing relays in the $n$-th subsequent ARQ after $D_0 \geq 1$.

Definition 3: Let $P_{D_{n,\ell}} \triangleq \sum_{\ell=0}^{n} D_{\ell}$.

The probability $P_{D_{n,\ell}}$ can be obtained by setting $D_0 = D$ in (8). In addition, to characterize the outage probability of overhearing, we define $V_{m,\ell} \triangleq \eta |h_{r_m,r_{\ell}}|^2 \sim \text{Exp}(\lambda_3)$ with $\lambda_3 \triangleq 1/(\eta)$ and $h_{r_m,r_{\ell}}$ standing for the channel between the transmit relay $m \in S_D$ and the receive relay $\ell \notin S_D$. Since the channels from relays to relays are random, the outage probability for a relay to overhear the ODSTCi signal sent by relays in $S_D$ involves no channel ordering effect and is thus given by
\[
P_{V|D}(\delta|n+1) \triangleq P \left\{ V \triangleq \sum_{m=1}^{n+1} V_{m,\ell} < \delta \right\}
\]
\[= 1 - \sum_{y=0}^{n+1} \frac{1}{y!} (\frac{1}{\lambda_3})^y
\]
where $n_{i+1} \triangleq \min\{i, d_n\}$. Moreover, as the source stops sending signal once $D_0 \geq 1$, given (19), we then have
\[
P_{D_{n,\ell}}(d_n|d_{n-1}) = C_{d_n}^{M-d_{n-1}} [1 - P_{V|D}(\delta|n)]^{d_n} \times
\]
\[P_{V|D}(\delta|n)^{M-d_n-d_{n-1}, n = 1, 2, \ldots}
\]

With $P_{D_{n,\ell}}$ and $P_{\text{C},i,\ell}$, the outage probability for the Type-C ARQ protocol can now be derived easily by induction. The result is summarized in the following proposition.

Proposition 2: Given $R$, $\varepsilon$ and $M$, the outage probability after $n$ times Type-C ARQs for ODSTCi is given by
\[
P_{\text{C},i}(n) = P_{W}^{n+1}(\delta_i) P_{D}(0) +
\]
\[
P_{W}(\delta_i) \sum_{k=1}^{n} [P_{W}(\delta_i) P_{D}(0)]^{n-k} \sum_{d_{n-1}=0}^{M-d_{n-2}} \cdots \sum_{d_{k}=0}^{M-d_{k-2}} \sum_{d_{k-1}=0}^{M-d_{k-1}}
\]
\[P_{\text{C},i,\ell}(\delta|d_\ell) P_{D}(d_\ell) \prod_{t=1}^{k-1} P_{\text{C},i,\ell}(\delta|d_t) P_{D}(d_t|d_{t-1})
\]
\[\forall i \in [1, M] \text{ with } P_{\text{C},i}(0) \triangleq P_{W}(\delta_i).
\]

With $d_0 \triangleq \sum_{n=0}^{n} d_n$ and $P_{\text{C},i}(0) \triangleq P_{W}(\delta_i)$.

As we can see from (6) that the diversity order of $P_{\text{C},i}(\delta|d_\ell)$ is $d_\ell$, and from (20) that the diversity order of $P_{\text{D},i,\ell}(d_\ell|d_{\ell-1})$ is $(M - d_\ell) \times \min\{i, d_{\ell-1}\}, \ell \geq 1$. Considering the dominating term only, which has minimum diversity order, we may find that (21) is dominated by the second term when $k = n$, $d_0 = 1$ and $d_1 = \ldots = d_{n-1} = 0$, leading to a diversity order of $nM + 1$. This shows that the diversity can increase by $M$ with each extra ARQ due to the aid of overhearing. Nevertheless, the complexity of this protocol is much higher too as all relays need to serve a user until the end of ARQs. On the other hand, for the Type-A protocol, even if the diversity order is inferior, the active relays are fixed in all subsequent ARQs, thus introducing less control overhead to the theatre system. It’s a tradeoff between the theoretical performance and practical considerations. Therefore, an investigation on the expected throughput of each protocol will be done in the next section to examine the efficiency of each protocol in different channel conditions.

V. THROUGHPUT ANALYSIS

Based on the outage analysis, we further study in this section the effective throughput of the three types of ARQ protocols. This analysis can help provide a systematic view and better understanding on the use of ODSTCi scheme for ARQs.

We consider transmitting a packet of source data of duration $T_s$ to the destination. Given the code rate $R$, the total amount of source information carried by the packet is $RT_s$. Therefore, the relays forward the signal, if necessary, with ODSTCi at a rate of $\varepsilon R$, the packet duration of relaying becomes $T_s/\varepsilon$.

In addition to the code rate, the throughput is also affected by the acquisition delay of ARQ. However, the acquisition delay depends on the system design as well as the data traffic within the system. To conduct a more general analysis, we express the average delay of ARQ to the source as $T_s/\varepsilon$, and the average delay of ARQ to the relays as $T_s + T_s$, where $T_s,T_s \in R$ and both $T_s,T_s$ includes the transmission time and the acquisition delay.

Now, denote the average throughput in a maximum of $n$ times of ARQs by $\gamma_T(i, n)$, where the subscript, $T \in \{A,B,C\}$, is used to distinguish from the different types of ARQ protocols and $i$ stands for the maximum number of active relays for ODSTCi. Given that $n_s + n_r \leq n$ and $n_s, n_r, \geq 0$, the average throughput can be expressed as
\[
\gamma_T(i, n) = \sum_{n_s=0}^{n-n_r} \sum_{n_r=0}^{n-n_s} \frac{T_s}{T_s + n_sT_s + n_rT_s} \cdot \frac{1}{\varepsilon}
\]
\[\forall i \in [1, M] \text{ with } P_{\text{C},i}(n_s, n_r).
\]

The probability, $P_{\text{T},i}(n_s, n_r)$, of the successful packet delivery also depends on $R$, $i$ and the types of protocols, $T$.

Proposition 3: For the three types of ARQ protocols defined in Section IV, the probability of a successful packet delivery, $P_{\text{T},i}(n_s, n_r)$, with $n_s$ times of ARQs through source BC followed by $n_r$ times of ARQs via relay forwarding using ODSTCi are given by
\[1. \text{ If } n_r = 0, \text{ then}
\]
\[P_{\text{T},i}(n_s, 0) = [P_{W}(\delta_i) P_{D}(0)]^{n_s} [1 - P_{W}(\delta_i)].
\]
2. If \( n_r = 1 \), then
\[
\mathbf{P}_{T,i}(n_s,1) = P_W(\delta_s)[P_W(\delta_s)\mathbf{P}_D(0)]^{n_s} \times \sum_{d=1}^{M} \mathbf{P}_D(d)[1 - P_{\mathcal{O}_1}(\delta|d)].
\] (24)

3. For \( n_r \geq 2 \), we have
\[
\mathbf{P}_{A,i}(n_s,n_r) \triangleq P_W(\delta_s)[P_W(\delta_s)\mathbf{P}_D(0)]^{n_s} \times \sum_{d=1}^{M} \mathbf{P}_D(d)P_{\mathcal{O}_1}(\delta|d)\mathbf{P}_{\mathcal{O}_2}(\delta|d)^{n_r-2}(\delta|d)[1 - P_{\mathcal{O}_1}(\delta|d)].
\] (25)
\[
\mathbf{P}_{B,i}(n_s,n_r) \triangleq P_W(\delta_s)[P_W(\delta_s)\mathbf{P}_D(0)]^{n_s} \times \sum_{d=1}^{M} \mathbf{P}_D(d)P_{\mathcal{O}_1}(\delta|d)[1 - P_{\mathcal{O}_1}(\delta|d)].
\] (26)

and
\[
\mathbf{P}_{C,i}(n_s,n_r) \triangleq P_W(\delta_s)[P_W(\delta_s)\mathbf{P}_D(0)]^{n_s} \times \sum_{d_1=1}^{M-d_0} \sum_{d_2=0}^{M-d_0-2} \cdots \sum_{d_n-1=0}^{M-d_0-2} \mathbf{P}_{\mathcal{O}_1}(\delta|d_1)\mathbf{P}_{\mathcal{O}_2}(\delta|d_2)\cdots\mathbf{P}_{\mathcal{O}_n}(\delta|d_n-1)\left[1 - P_{\mathcal{O}_1}(\delta|d_n-1)\right].
\] (27)

Proof: The proof is omitted for space.

VI. Numerical Simulations

To examine the delay-limited throughput for different types of ARQ protocols, we adjust the rate according to the SNR to achieve the best throughput subject to an outage constraint \( P_e \), i.e.
\[
\max_{R} \zeta_{T,i}(n) \quad \text{s.t.} \quad \mathbb{P}_{T,i}(n) \leq P_e.
\] (28)

Given the feedbacks of the source-destination and relay-destination link qualities, \( \rho \) and \( \beta \rho \), the transmission rates for the source and the relays can be expressed as \( R \triangleq \gamma \log(1 + \rho) \) and \( \varepsilon R \triangleq \gamma \log(1 + \beta \rho) \), respectively. As a result, the constrained throughput maximization problem in (28) can be reformulated as
\[
\max_{\gamma} \zeta_{T,i}(n) \quad \text{s.t.} \quad \mathbb{P}_{T,i}(n) \leq P_e.
\] (29)

This maximization problem can be easily solved with the typical steepest descent algorithm. We next present the optimal throughput for the three types of ARQ protocols under different system settings in practical ranges of SNRs.

We consider a channel condition with \( \alpha = 8 \) and \( \beta = 2 \). This case is meaningful, especially when the direct link between the source and the destination has a low SNR, e.g. \( 0 \sim 10 \text{dB} \). Setting \( \beta = 2 \) gives the link quality between the relays and the destination a 3dB-gain over the direct link. And \( \alpha = 8 \) gives the source-relay link a 9dB-gain over the direct link.

Conditioned on the delay constraint \( n = 3 \) and the outage constraint \( P_e = 0.001 \), Fig. 4 presents the maximum throughput for each type of ARQ protocol with ODSCT2 when \( M = 5, \alpha = 8, \beta = 2 \) and \( \eta = 512 \). The purple curve denotes the simple ARQs without relaying. Clearly, the cooperative ARQ protocols provide significant throughput improvement not only when the link quality between the source and destination has good SNR, but also when the link quality is poor, e.g. SNR = 0 \sim 10\text{dB}. The Type-C ARQ gives the highest throughput due to the overhearing and relay re-selection mechanisms involved in the protocol.

Nevertheless, in low or high SNR regimes, Fig. 4 also shows that the differences between the ARQ protocols are getting smaller. The reason is that the relays and the destination tend to fail or succeed in decoding altogether in these two regimes. Thus, there is no need to always use the most complicated scheme even if it has the superior diversity in outage analysis.

On the other hand, the effects of the acquisition delays of ARQs are represented in Fig. 5. Larger \( \tau_r \) correspond to longer delays for relays to access the channel, and \( \tau_s = 1 \) stands for no acquisition delay. As shown in the figure, \( \tau_s \) introduces serious throughput degradation. However, \( \tau_s \) does not have significant influence on the throughput, comparing the cases of \( \tau_s = \tau_r = 1 \) and \( \tau_s = 10, \tau_r = 1 \).

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it is not always necessary to use as many relays as possible. The throughput of $i=2$ is almost indistinguishable from that of $i=3$ in Fig. 6. In other words, ODSTC2 is an effective scheme under such channel condition. In particular, it can be easily implemented with the Alamouti STC. Moreover, if the channel quality is very good, i.e., when SNR > 15dB, ODSTC1 gives almost the same performance as if using all available relays.

Fig. 7 shows the effect of $M$ on the throughput of different ARQ protocols with ODSTC2 at SNR = 10dB. In contrast to the Type-A scheme, the advantage of Type-B ARQ increases with $M$. This is due to the fact that the Type-B scheme has better coding gains from the re-selection mechanism, although they have the same diversity order. Besides, the performance of the Type-C scheme will saturate too even with a larger $M$ since a ultra high diversity order does not lead to a comparative enhancement in throughput. This result can be very helpful for multiple access systems as only a limited number of relays is needed to assign to each user.

VII. Conclusion

The numerical results showed that the cooperative ARQ protocols provide significant throughput enhancement in contrast to the direct transmission without relaying. Besides, effective schemes can be obtained since it is not always necessary to use the most complicated protocol or as many relays as possible to achieve the best performance. With these advantages of ODSTC relaying, more advanced cooperative ARQ protocols may be developed for multiple access systems to enhance the overall system throughput with assigning only a limited number of relays to each user.

References