MIMO Wireless Communications

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Outline

- MIMO wireless channels
- MIMO transceiver
- MIMO precoder
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Characteristics of Wireless Channels

- **Main characteristics of wireless channels**
  - The channel strength varies with time, frequency and space

- **Large-scale fading:**
  - Path loss: a function of distance
  - Shadowing: due to large objects between transmitter and receiver

- **Small scale fading:**
  - Multipath effect: interferences of multiple signal paths
  - Doppler effect: relative speed between transmitter and the receiver
A multiple input multiple output (MIMO) system with $M$ transmit elements and $N$ receive elements.
MIMO Channel Model

For the above MIMO channel, the baseband input-output relationship can be expressed as

\[ y(t) = H(t) * s(t) + n(t) \]

Where \( H(t) \) is a \( N \) by \( M \) channel impulse response matrix.

If the signal bandwidth is sufficiently narrow such that the channel can be treated as approximately constant over the operating frequency (frequency flat channel), the corresponding input-output relationship simplifies to

\[ y(t) = H(t)s(t) + n(t) \]

Where \( H \) is the narrowband MIMO channel matrix.
MIMO Channel Modeling

- The modeling of the channel impulse response $H(t)$ or channel matrix $H$ is critical for the simulation of the MIMO communication systems.

- Due to insufficient spacing between antenna elements and limited scattering in the environment, the elements of the channel matrix are not always independent.

- When modeling the MIMO channels, these effects should be taken into account.
Several classifications of the MIMO channel models are given below

2. Wideband Models vs. Narrowband Models

**Wideband Models**
- The wideband models treat the propagation channel as frequency selective; different frequency sub-bands have different channel responses

**Narrowband Models**
- The narrowband models assume that the channel has frequency non-selective fading and thus the channel has the same response over the entire system bandwidth
1. Field Measurements vs. Scatterer Models

**Field Measurements**
- Measure the MIMO channel responses through field measurements
- Some important characteristics of the MIMO channel can be obtained by investigating the recorded data

**Scatterer Models**
- Postulating a model (usually involving distributed scatterers) to capture the channel characteristics
- As long as the constructed scattering environment is reasonable, such a model can often capture the essential characteristics of the MIMO channel
Model Classification

1. Non-physical Models vs. Physical Models

**Non-physical Models**
- The non-physical models are derived from the statistical characteristics of the MIMO channel
- They are easy to simulate but give limited insight to the propagation characteristics of the MIMO channels

**Physical Models**
- The physical models choose some crucial physical parameters (AOA, AOD, TOA, etc.) to describe the MIMO channels
- Such a model provides reasonable description of the MIMO channel characteristics and the surrounding scattering environment

- An indoor measurement campaign carried out in Aalborg, Denmark at a carrier frequency of 2.05 GHz.
- A stochastic model for non-line-of-sight (NLOS) scenarios
- Based on the power correlation matrix of the MIMO radio channel

Let $M$ be the number of transmit antennas and $N$ be the number of receive antennas. In the proposed wideband model, the MIMO channel without noise is expressed as

$$H(\tau) = \sum_{l=1}^{L} H_l \delta(\tau - \tau_l),$$

where $H(\tau)$ is the $N \times M$ matrix of channel impulse responses.
**METRA MIMO Channel Model**

$H_l$ is the matrix of complex channel coefficients at time delay $\tau_l$

$$H_l = \begin{bmatrix}
H_{11}^l & H_{12}^l & \cdots & H_{1M}^l \\
H_{21}^l & H_{22}^l & \cdots & H_{2M}^l \\
\vdots & \vdots & \ddots & \vdots \\
H_{N1}^l & H_{N2}^l & \cdots & H_{NM}^l 
\end{bmatrix}$$

Assume

- Coefficients are zero mean complex Gaussian
- The same average power $p_l$ is assumed for the coefficients
- Coefficients are independent from one time delay to another
METRA MIMO Channel Model

- The correlation between different pairs of complex transmission coefficients need to be taken into account.

The spatial power correlation coefficients at the transmitter:

\[ \rho_{m_1m_2}^{Tx} = \langle |H_{nm_1}^l|^2, |H_{nm_2}^l|^2 \rangle \]

The spatial power correlation coefficients at the receiver:

\[ \rho_{n_1n_2}^{Rx} = \langle |H_{n_1m}^l|^2, |H_{n_2m}^l|^2 \rangle \]

It is claimed that the spatial cross correlation coefficients \cite{1}:

\[ \rho_{n_1m_1}^{n_2m_2} = \langle |H_{n_1m_1}^l|^2, |H_{n_2m_2}^l|^2 \rangle = \rho_{m_1m_2}^{Tx} \rho_{n_1n_2}^{Rx} \]

Where:

\[ \rho = \langle a, b \rangle = \frac{E[ab] - E[a]E[b]}{\sqrt{(E[a^2] - E[a]^2)(E[b^2] - E[b]^2)}} \]
In matrix form this can be written as

\[ P_H = P_H^{Tx} \otimes P_H^{Rx} \]

- \( P_H \): The power correlation matrix of the MIMO channel
- \( P_H^{Tx} \): The power correlation matrices seen from the transmitter
- \( P_H^{Rx} \): The power correlation matrices seen from the receiver
- \( \otimes \): The Kronecker product

Given \( P_H \)

\[ \text{vec}(H_l) = \sqrt{p_l} C a_l \]

\[ a_l \sim \mathcal{CN}(0, I) : MN \times 1 \]

\[ [CC^T]_{i,j} = [P_H]_{i,j}^{1/2} \]

- \( p_l \) is determined by the power delay profile
One drawback of the above model is that the phase relationship between transmission coefficients is lost.

So it was suggested in [2] to multiply a phase steering diagonal matrix $W(\bar{\phi}_{Rx})$ after the convolution between the MIMO channel impulse response and the transmitted signal.

The received signal without noise can be written as

$$y(t) = W(\bar{\phi}_{Rx}) \int H(\tau)s(t - \tau)d\tau$$

where the diagonal elements of $W(\bar{\phi}_{Rx})$ provide the average phase shift information relative to the first receive element and $\bar{\phi}_{Rx}$ is the mean azimuth AOA.
**EU IST SATURN (Smart Antenna Technology in Universal bRoadband wireless Network) Project**

- An indoor measurement campaign carried out in Bristol
- For non-line-of-sight (NLOS) scenarios
- Based on the first and second order moments of the measured data
- Let $M$ be the number of transmit antennas and $N$ be the number of receive antennas
- It was found [3] that in the typical NLOS scenarios, the channel coefficients are zero mean complex Gaussian
- Furthermore, it was reported that the channel covariance matrix can be well approximated by the Kronecker product of the covariance matrices seen from both ends
SATURN MIMO Channel Model

\[ \mathbf{R}_H = \mathbf{R}_H^{Tx} \otimes \mathbf{R}_H^{Rx} \]

\[ \mathbf{R}_H^{Tx} = \mathbb{E}[(\mathbf{h}_i^H \mathbf{h}_i)^T], \quad \text{for } i = 1, \ldots, N \]

\[ \mathbf{R}_H^{Rx} = \mathbb{E}[\mathbf{h}_j^H \mathbf{h}_j^H], \quad \text{for } j = 1, \ldots, M \]

where \( \mathbf{h}_i \) is the \( i \)-th row of \( \mathbf{H} \), \( \mathbf{h}_j \) is the \( j \)-th column of \( \mathbf{H} \) and \((\ )^H\) is complex conjugate transpose.

It is easy to show that

\[ \mathbf{H} = (\mathbf{R}_H^{Rx})^{1/2} \mathbf{G}(\mathbf{R}_H^{Tx})^{T/2} \]

where \( \mathbf{G} \) is a stochastic \( N \) by \( M \) matrix with IID \( \mathcal{C}\mathcal{N}(0,1) \) elements and \((\ )^{1/2}\) denotes any matrix square root such that \( \mathbf{R}^{1/2}(\mathbf{R}^{1/2})^H = \mathbf{R} \)
Non-Physical MIMO Channel Models

Compare METRA Project with SATURN Project

- It can be seen that the expression in SATURN Project is very close to the model in METRA Project

- In SATURN Project, the channel covariance matrix is used instead of the power correlation matrix in METRA Project

- Therefore SATURN Project provides the phase information of the MIMO propagation channel

- The structure in SATURN Project was also discussed in the 3GPP (3rd Generation Partnership Project) meeting
One-ring model

- The base station (BS) is not obstructed by local scattering while the mobile station (MS) is surrounded by scatterers.
- No line-of-sight (LOS) is assumed between the BS and MS.
- \( T_p \) is the pth antenna element at the BS, \( R_n \) is the nth antenna element at the MS.
- \( D \) is the distance between the BS and MS, \( R \) is the radius of the ring of scatterers.
Denote the effective scatterer on the ring by $S(\theta)$ and let $\theta$ be the angle between the scatterer and the array at the MS.

In the model, it is assumed that $S(\theta)$ is uniformly distributed in $\theta$.

The phase shift, $\phi(\theta)$, associated with each scatterer, $S(\theta)$, is distributed uniformly over $[-\pi, \pi)$ and IID in $\theta$.

Each ray is further assumed to be reflected only once.

All rays reach the receive array with the same power.
One-Ring MIMO Channel Model

- Suppose there are $K$ effective scatterers $S(\theta_k)$, $k = 1, 2, \ldots, K$ distributed on the ring.

- The complex channel coefficient between the $p$-th elements at the BS and $n$-th element at the MS can be expressed as

  \[ H_{p,n} = \frac{1}{\sqrt{K}} \sum_{k=1}^{K} \exp\{-j \frac{2\pi}{\lambda} (D_{Tp \rightarrow S(\theta_k)} + D_{S(\theta_k) \rightarrow Rn} + j \phi(\theta_k))\} , \]

  where $D_{X \rightarrow Y}$ denotes the distance between $X$ and $Y$.

  $\lambda$ is the wavelength.

- The covariance between $H_{p,n}$ and $H_{q,m}$ is given by

  \[
  E[H_{p,n} H_{q,m}^H] = \frac{1}{K} \sum_{k=1}^{K} \exp\{-j \frac{2\pi}{\lambda} (D_{Tp \rightarrow S(\theta_k)} - D_{Tq \rightarrow S(\theta_k)} + D_{S(\theta_k) \rightarrow Rn} - D_{S(\theta_k) \rightarrow Rm})\} . (20)
  \]
Two-Ring MIMO Channel Model

**Two-ring model**

- The two-ring model assumes that both the BS and MS are surrounded by scatterers
- Each ray is reflected twice
- This can be the case for indoor wireless communications
Two-Ring MIMO Channel Model

- The channel coefficient for the two-ring model is

\[ H_{p,n} = \frac{1}{\sqrt{K_1 K_2}} \sum_{k=1}^{K_1} \sum_{l=1}^{K_2} \exp\{-j \frac{2\pi}{\lambda} (D_{T \rightarrow S_1(\alpha_k)} + D_{S_1(\alpha_k) \rightarrow S_2(\beta_l)} + D_{S_2(\beta_l) \rightarrow Rn}) \} + j \phi_1(\alpha_k) + j \phi_2(\beta_l) \}. \]  

(21)

- The difficulty in this model is that the signals reflected by the scatterers at the receive side are possibly not independent.
- Even if the numbers of scatterers, \(K_1\) and \(K_2\) go to infinity, the channel coefficient is still not zero mean complex Gaussian.
- Therefore, the channel covariance matrix can not completely describe the MIMO channel.
Distributed Scattering model

- This narrowband model was proposed to describe outdoor MIMO propagation channels

- Assume there are $M$ transmit elements and $N$ receive elements

- Both the transmitter and receiver are obstructed by the surrounding scatterers
MIMO Channel Modeling

- Assume there are \( S \) scatterers on both the transmitter and receiver.
- The scatterers at the receive side can be seen as a virtual array between the transmitter and receiver.

**The MIMO channel transfer function is given by**

\[
H = \frac{1}{\sqrt{S}} R_{\theta_r, d_r}^{1/2} G_r R_{\theta_s, 2D_r / S}^{1/2} G_t R_{\theta_t, d_t}^T/2
\]

- \( 1/\sqrt{S} \) is a normalization factor.
- \( G_t \) (S by M) and \( G_r \) (N by S) are random matrices with IID zero mean complex Gaussian elements.
- \( R_{\theta_t, dt} \), \( R_{\theta_s, 2Dr / S} \), \( R_{\theta_r, dr} \) are the correlation matrices seen from the transmitter, virtual array and receiver respectively.
For uniformly distributed AOAs, the \((m,k)\)th element of the correlation matrix can be expressed as

\[
[R_{\theta,d}\theta,m,k]_{m,k} = \frac{1}{S} \sum_{i=-\frac{S-1}{2}}^{\frac{S-1}{2}} e^{-2\pi j(k-m)d \cos\left(\frac{\pi}{2} + \theta_i\right)}
\]

Where

- \(S\) should be odd
- \(d\) is the array element distance
- \(\theta_i\) is the AOA of the \(i\)th scatterer
3GPP Spatial Channel Model (SCM)

- From 3GPP TR 25.996 version 6.1.0 Release 6

- The 3GPP SCM is an outdoor channel model for a 5MHz bandwidth CDMA system in the 2GHz band

- It defines three environments (Suburban Macro, Urban Macro, and Urban Micro)

- There is a fixed number of 6 “paths” in every scenario and each is made up of 20 spatially separated “sub-paths”

- Path powers, path delays, and angular properties for both sides of the link are modeled as random variables
These 6 multipaths are defined by powers and delays and are chosen randomly according to the channel generation procedure. Each path consists of 20 subpaths.

Figure 5.2: BS and MS angle parameters
SCM MIMO Channel Model

- SCM for simulations

1. Choose scenario
   - Suburban macro
   - Urban macro
   - Urban micro

2. Determine user parameters
   - Angle spread $\sigma_{AS}$
   - Lognormal shadowing $\sigma_{LN}$
   - Delay spread $\sigma_{DS}$
   - Pathloss
   - Orientation, Speed Vector $\theta_{BS} \theta_{MS} \Omega_{MS}$
   - Antenna gains
   - $\delta_{n,AoD}$ Angles of departure (paths)
   - $\Delta_{n,m,AoD}$ Angles of departure (subpaths)
   - $\tau_n$ Path delays
   - $P_n$ Average path powers
   - $\delta_{n,AoA}$ Angles of arrival (paths)
   - $\Delta_{n,m,AoA}$ Angles of arrival (subpaths)

3. Generate channel coefficients
   - Polarization
   - LOS (urban micro)

Figure 5.1: Channel model overview for simulations
SCM MIMO Channel Model

- **Spatial parameters for the BS and the MS**
  
  **Array topologies**
  
  - Antenna spacing
    
    **MS**: the reference element spacing is $0.5\lambda$
    
    **BS**: three values for reference element spacing are defined: $0.5\lambda$, $4\lambda$, and $10\lambda$

- **Per-path angle spread (AS)**
  
  - The BS (or MS) per-path angle spread is defined as the root mean square (RMS) of angles with which an arriving (or incident) path’s power is received by the BS (or MS) array
SCM MIMO Channel Model

**BS:**
- AS = 2 degrees at AoD 50 degrees
- AS = 5 degrees at AoD 20 degrees

**MS:**
- AS = 104 degrees (results from a uniform over 360 degree PAS)
- AS = 35 degrees for a Laplacian PAS with a certain path specific Angle of Arrival (AoA)

**Per-path AOA/AOD**

**BS:**
- AoD: 50 degrees (with the RMS AS of 2 degrees)
- AoD: 20 degrees (with the RMS AS of 5 degrees)

**MS:**
- AOD : -67.5, +67.5, +22.5 degrees (with the RMS AS of 2 degrees)
Per-path power azimuth spectrum

BS :
The Power Azimuth Spectrum (PAS) of a path arriving at the base station is assumed to have a Laplacian distribution

MS :
The Laplacian distribution and the Uniform distribution are used to model the per-path PAS at the MS
General assumptions and parameters

- For an $S$-element BS array and a $U$-element MS array, the channel is given by an $U \times S$ matrix of complex amplitudes.

- Denote the channel matrix for the $n$-th multipath component ($n = 1, \ldots, 6$) as $H_n(t)$.

$$H_n(t) = \begin{bmatrix} h_{1,1,n} & \cdots & h_{1,s,n} \\ \vdots & \ddots & \vdots \\ h_{u,1,n} & \cdots & h_{u,s,n} \end{bmatrix}$$
These 6 multipaths are defined by powers and delays and are chosen randomly according to the channel generation procedure. Each path consists of 20 subpaths.

Figure 5.2: BS and MS angle parameters
SCM MIMO Channel Model

Figure 5.2 shows the angular parameters used in the model. The following definitions are used:

\[ \Omega_{BS} \]
BS antenna array orientation, defined as the difference between the broadside of the BS array and the absolute North (N) reference direction.

\[ \theta_{BS} \]
LOS AoD direction between the BS and MS, with respect to the broadside of the BS array.

\[ \delta_{n,AoD} \]
AoD for the nth \((n = 1 \ldots N)\) path with respect to the LOS AoD \(\theta_0\).

\[ \Delta_{n,m,AoD} \]
Offset for the mth \((m = 1 \ldots M)\) subpath of the nth path with respect to \(\delta_{n,AoD}\).

\[ \theta_{n,m,AoD} \]
Absolute AoD for the mth \((m = 1 \ldots M)\) subpath of the nth path at the BS with respect to the BS broadside.

\[ \Omega_{MS} \]
MS antenna array orientation, defined as the difference between the broadside of the MS array and the absolute North reference direction.

\[ \theta_{MS} \]
Angle between the BS-MS LOS and the MS broadside.

\[ \delta_{n,AoA} \]
AoA for the nth \((n = 1 \ldots N)\) path with respect to the LOS AoA \(\theta_{0,MS}\).

\[ \Delta_{n,m,AoA} \]
Offset for the mth \((m = 1 \ldots M)\) subpath of the nth path with respect to \(\delta_{n,AoA}\).

\[ \theta_{n,m,AoA} \]
Absolute AoA for the mth \((m = 1 \ldots M)\) subpath of the nth path at the MS with respect to the BS broadside.

\[ v \]
MS velocity vector.

\[ \theta_v \]
Angle of the velocity vector with respect to the MS broadside: \(\theta_v = \text{arg}(v)\).
SCM MIMO Channel Model

Generating channel coefficients

Given the user parameters generated, we use them to generate the channel coefficients.

The \((u,s)\)th component \((s = 1,...,S; u = 1,...,U)\) of \(H_n(t)\) is

\[
h_{u,s,n}(t) = \sqrt{\frac{P_n \sigma_{SF}}{M}} \sum_{m=1}^{M} \left( \frac{\sqrt{G_{BS} \left( \theta_{n,m,AoD} \right)}}{\sqrt{G_{MS} \left( \theta_{n,m,AoA} \right)}} \exp(j kd_s \sin(\theta_{n,m,AoD} + \Phi_{n,m})) \times \right.
\]

\[
\left. \frac{\sqrt{G_{MS} \left( \theta_{n,m,AoA} \right)}}{\sqrt{G_{BS} \left( \theta_{n,m,AoD} \right)}} \exp(j kd_u \sin(\theta_{n,m,AoA})) \times \exp(j \|v\| \cos(\theta_{n,m,AoA} - \theta_v) t) \right)
\]

\[
\theta_{n,m,AoD} = \theta_{BS} + \delta_{n,AoD} + \Delta_{n,m,AoD}
\]

\[
= \theta_{n,AoD} + \Delta_{n,m,AoD}
\]

\[
\theta_{n,m,AoA} = \theta_{MS} + \delta_{n,AoA} + \Delta_{n,m,AoA}
\]

\[
= \theta_{n,AoA} + \Delta_{n,m,AoA}
\]
Spatial cross-correlation function (CCF) for SCM

To calculate the CCF, we use a simplified version of the expression $h_{u,s,n}(t)$ by neglecting the shadowing factor $\sigma_{SF}$ and assuming that the antenna gains of each array element $G_{BS}(\theta_{n,m,AoD}) = G_{MS}(\theta_{n,m,AoA}) = 1$

The normalized complex spatial temporal correlation function between two arbitrary channel coefficients connecting two different sets of antenna elements is defined as

$$
\rho_{s_1u_1}^{s_2u_2}(\Delta d_s, \Delta d_u, \tau) = E \left\{ \frac{h_{u_1,s_1,n}(t)h^*_{u_2,s_2,n}(t + \tau)}{\sigma_{h_{u_1,s_1,n}} \sigma_{h_{u_2,s_2,n}}} \right\}
$$
Substitution $h_{u,s,n}(t)$ into the correlation function

$$
\rho_{s_1u_1}^{s_2u_2}(\Delta d_s, \Delta d_u, \tau) = \frac{1}{M} \sum_{m=1}^{M} E\{\exp[jk\Delta d_s \sin(\theta_{n,m}, AoD)] \\
\cdot \exp[-jk||\mathbf{v}|| \cos(\theta_{n,m}, AoA - \theta_v)\tau] \\
\cdot \exp[jk\Delta d_u \sin(\theta_{n,m}, AoA)]\} \tag{5}
$$

where $\Delta d_s = |d_{s_1} - d_{s_2}|$ and $\Delta d_u = |d_{u_1} - d_{u_2}|$ denote the relative BS and MS antenna element spacings, respectively.

For spatial CCFs, by imposing $\tau = 0$

$$
\rho_{s_1u_1}^{s_2u_2}(\Delta d_s, \Delta d_u) = \frac{1}{M} \sum_{m=1}^{M} E\{\exp[jk\Delta d_s \sin(\theta_{n,m}, AoD)] \\
\cdot \exp[jk\Delta d_u \sin(\theta_{n,m}, AoA)]\}. \tag{6}
$$
Some special cases:

i) $\Delta d_s = 0$, results in the spatial CCF observed at the MS

$$\rho_{u_1 u_2}^{MS} (\Delta d_u) = \frac{1}{M} \sum_{m=1}^{M} E\{\exp[jk\Delta d_u \sin(\theta_{n,m, AoA})]\} . \quad (7)$$

ii) $\Delta d_u = 0$, results in the spatial CCF observed at the BS

$$\rho_{s_1 s_2}^{BS} (\Delta d_s) = \frac{1}{M} \sum_{m=1}^{M} E\{\exp[jk\Delta d_s \sin(\theta_{n,m, AoD})]\} . \quad (8)$$

Note that the spatial CCF in (6) cannot simply be broken down into the multiplication of a receive term (7) and a transmit term (8). This indicates that the spatial CCF of the 3GPP SCM is in general not separable.
iii) $M \to \infty$, from (6)

$$\lim_{M\to\infty} \rho_{s_1 u_1}^{s_2 u_2} (\Delta d_s, \Delta d_u)$$

$$= \int_0^{2\pi} \int_0^{2\pi} \left\{ p_{us}(\phi_n, AoD, \phi_n, AoA) \exp[j k \Delta d_u \sin(\phi_n, AoA)] \cdot \exp[j k \Delta d_s \sin(\phi_n, AoD)] \right\} d\phi_n, AoD d\phi_n, AoA$$

(9)

where $p_{us}(\psi_{n,AoD}, \psi_{n,AoA})$ represents the joint probability density function (PDF) of the AoD and AoA.
(iv) $\Delta d_s = 0$ and $M \to \infty$: From (7), we have

$$\lim_{M \to \infty} \rho^{MS}_{u_1 u_2}(\Delta d_u) = \int_{0}^{2\pi} \exp[jk\Delta d_u \sin(\phi_n, AoA)] p_u(\phi_n, AoA) d\phi_n, AoA$$

(10)

where $p_u(\phi_n, AoA)$ stands for the PDF of the AoA.

(v) $\Delta d_u = 0$ and $M \to \infty$: From (8), we have

$$\lim_{M \to \infty} \rho^{BS}_{s_1, s_2}(\Delta d_s) = \int_{0}^{2\pi} \exp[jk\Delta d_s \sin(\phi_n, AoD)] p_s(\phi_n, AoD) d\phi_n, AoD$$

(11)

where $p_s(\phi_n, AoD)$ denotes the PDF of the AoD.
For temporal ACF, let $\Delta ds = 0$ and $\Delta du = 0$ in (5)

$$r(\tau) = \frac{1}{M} \sum_{m=1}^{M} E\{\exp[-j k ||\mathbf{v}|| \cos(\theta_{n,m,\text{AoA}} - \theta_v) \tau]\}$$

$$= \rho_{s_1u_1}^{s_2u_2}(0, 0, \tau).$$

(12)

Note that

The comparison of (5), (6), and (12) clearly tells us that the spatial temporal correlation function $\rho_{s_1u_1}^{s_2u_2}(\Delta ds, \Delta du, \tau)$ is not simply the product of the spatial CCF $\rho_{s_1u_1}^{s_2u_2}(\Delta ds, \Delta du)$ and the temporal ACF $r(\tau)$. Therefore, the spatial temporal correlation of the SCM is in general not separable as well.
References

